



Sensitivity Analyses: Theoretical Background, Use in the Safety Case, and Related Research Activities

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Outline

- Sensitivity Analysis (SA) methods and their capabilities
- Recent research at TUC
- Open issues

What do we mean by „sensitivity analysis“?

- (Saltelli et al., 2006) “Sensitivity analysis is the study of how the variation on the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation, and of how the given model depends upon the information fed into it.”

- (Bolado/Röhlig/Becker 2008) “... many different interpretations.
 - Output variable response to an increment in some of the inputs
 - Correlation between inputs and outputs
 - Monotonic relation between inputs and outputs
 - A more complex polynomial (or non-polynomial) relation
 - Specific relations between different regions of input parameters and output variables
 - Output distribution changes as a result of input distribution changes
 - Fractional contribution to the output variance”

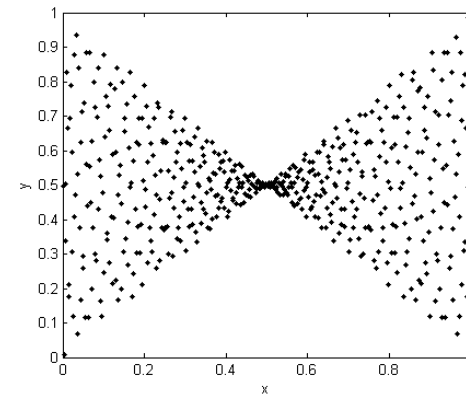
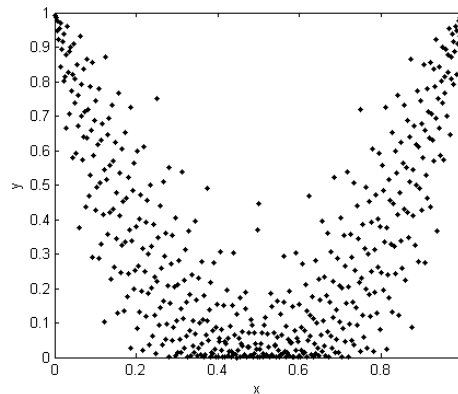
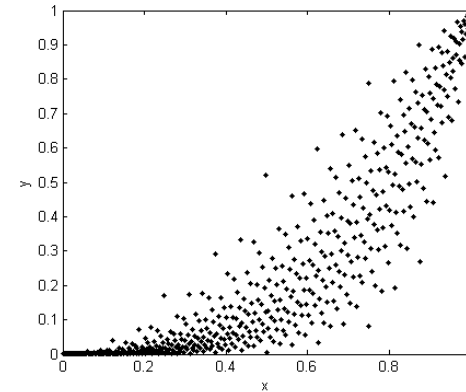
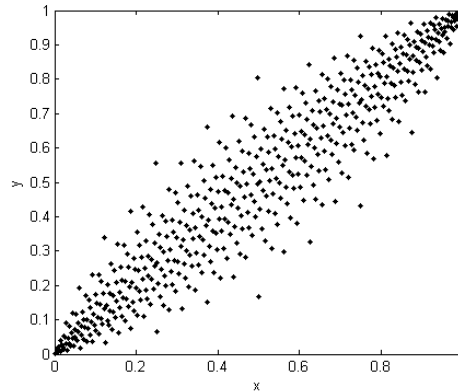
Sensitivity Analysis (SA) methods and their capabilities

- Due to time restrictions, the following is limited to so-called global SA
 - Probabilistic setting: Input factors with known distributions
 - Attempt to explore the whole input space

Examples of different global sensitivity methods

Consider the scatter plots on the right:

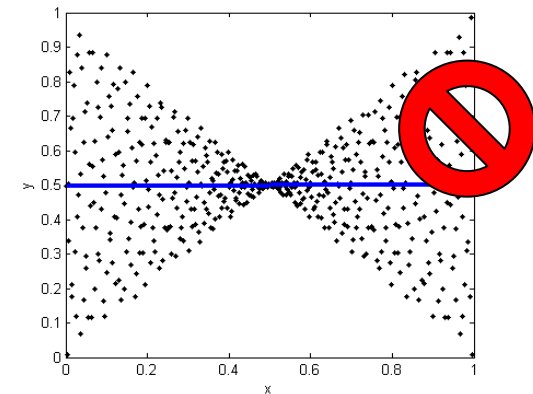
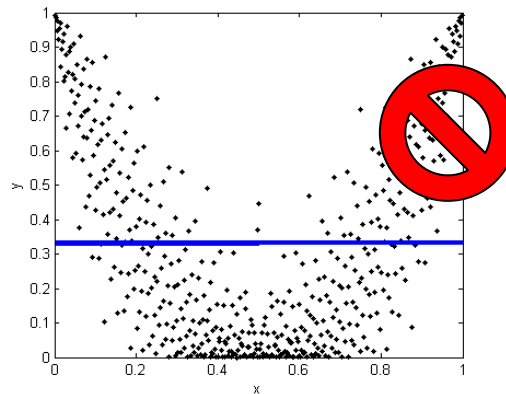
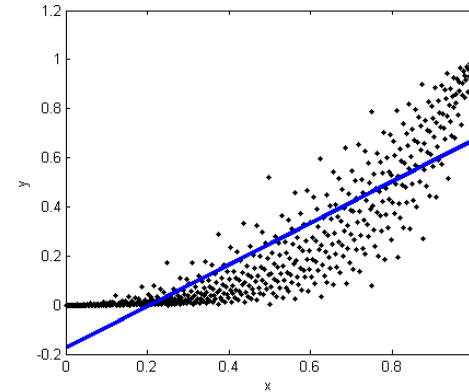
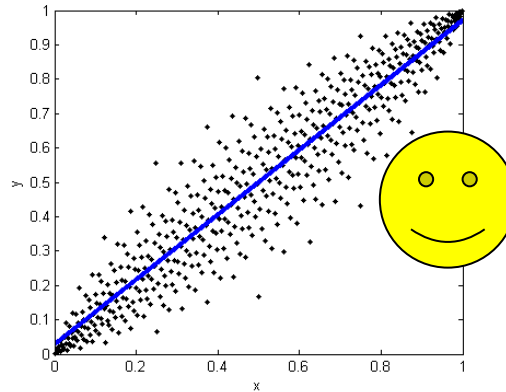
To which degree is the output y influenced by the input factor x ?



Examples of different global sensitivity methods

Linear regression:

How well*) is the output y explained by a linear model of the input factor x ?

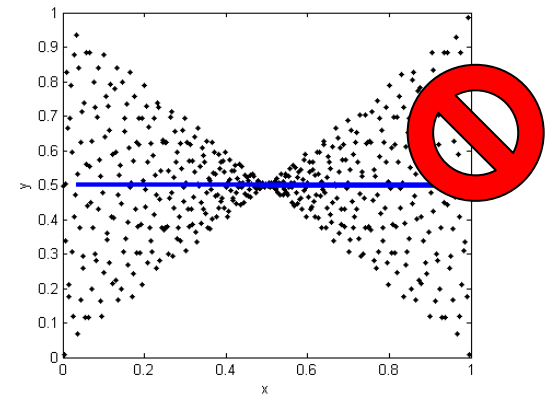
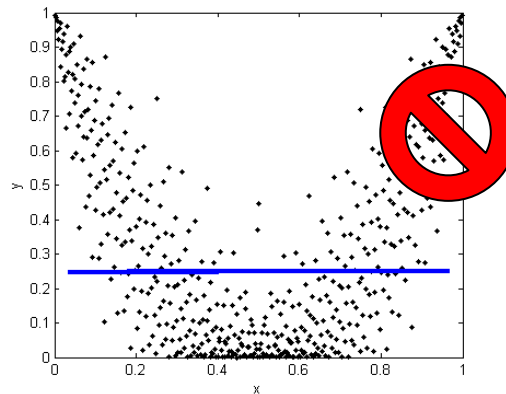
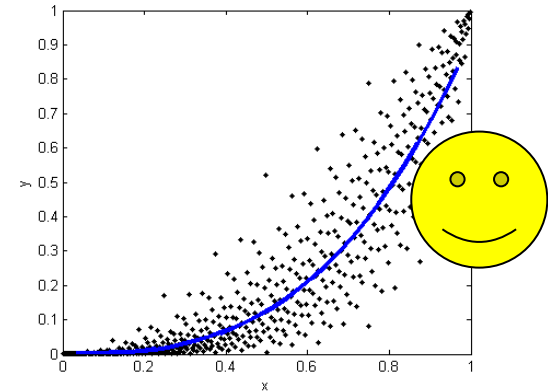
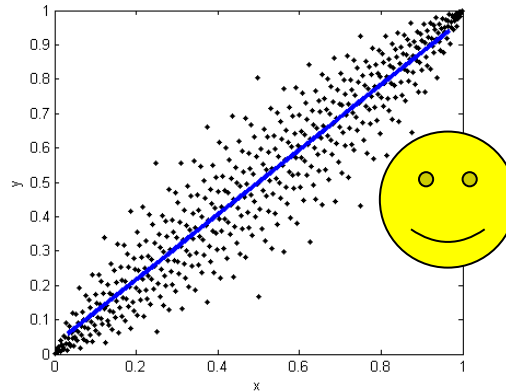


*) Goodness-of-fit/Coefficient-of-Determination used as Sensitivity indicator

Examples of different global sensitivity methods

Rank regression:

Monotonic trends in the data are picked up*)

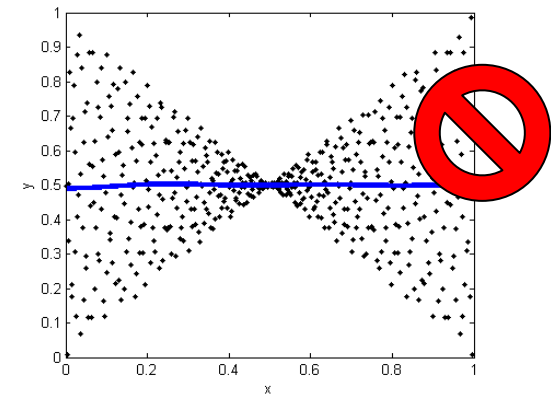
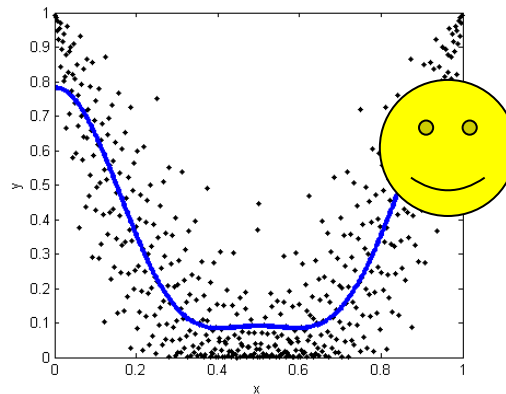
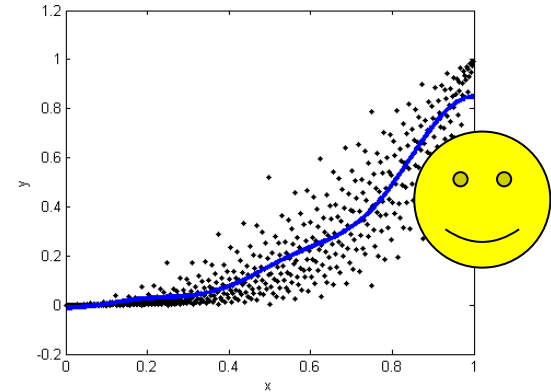
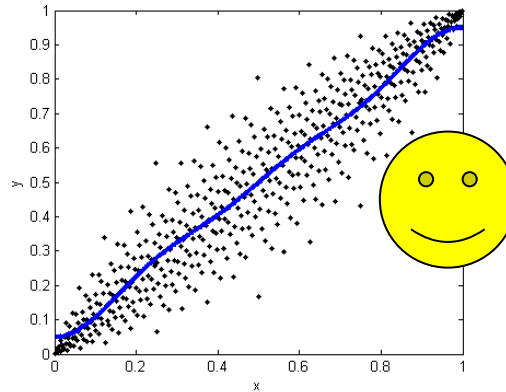


*) Goodness-of-fit/Coefficient-of-Determination used as Sensitivity indicator

Examples of different global sensitivity methods

Variance-based methods /
functional regression:

Functional dependencies
in the data are detected*)

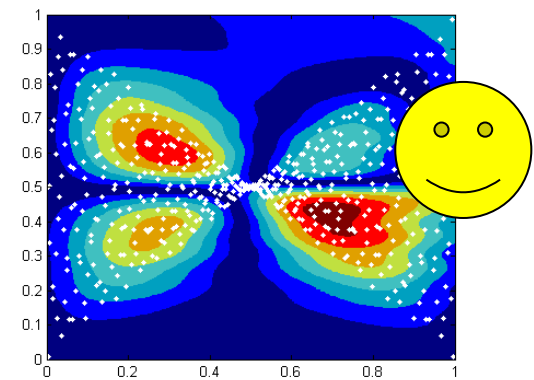
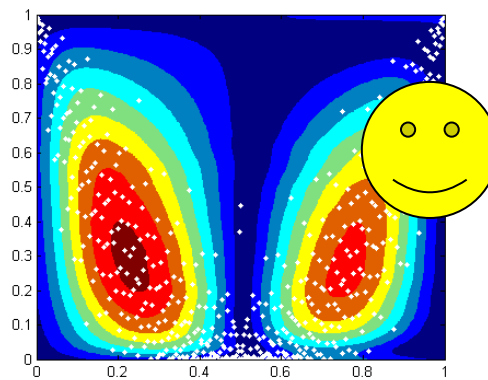
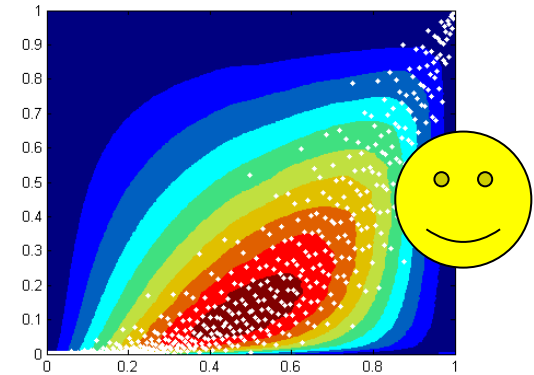
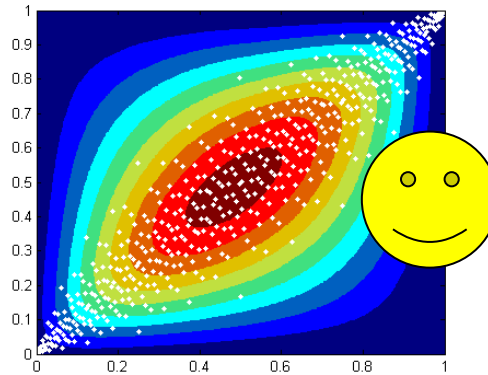


*) Goodness-of-fit/Coefficient-of-Determination used as Sensitivity indicator

Examples of different global sensitivity methods

Density-based methods:

Interaction terms /
statistical dependencies
in the data are
detected*)



*) No regression model,
distance between
conditional and
unconditional density as
sensitivity indicator

However ...

- Only linear and rank-based methods are routinely applied in most safety assessments in our field

- But:
 - What if established methods do not explain effects sufficiently well (indicated e.g. by $R^2 \ll 1$)?
 - What about interactions, non-monotonicity, time-dependency, discontinuities, ... ?
 - What about dependencies in input parameter set?

Current work at TUC (1): Theory

- Computationally efficient FAST-inspired Methods
 - Note that FAST (and others) work by associating frequencies to input parameters and then looking for resonances in output
 - This implies pre-defined sampling schemes, sample size considerably increasing with number of parameters
 - Methods **not** using such pre-defined schemes are of interest since they can considerably decrease computation costs

- Measuring distances between distributions

Current work at TUC (2): Application in PA

- „A suggestion to develop guidance on a general scheme for performing sensitivity analyses in safety assessments for geological disposal systems and interpreting results.” (MeSA conclusion)
- Testing SA with several (salt) models, issues in addition to those already mentioned (in co-operation with GRS):
 - Time dependency
 - Effects of Transformations on analysis results
 - Input dependencies
- Hitchhiker’s guide to sensitivity analysis in PA (in co-operation with GRS)

Hitchhiker's guide in a nutshell: Going beyond linear and rank-based methods

- Graphical ?
- Mean Ranks Plots
- Linear is $R^2 \approx 1$?
- Pearson Correlation
- Rank-based is $R^{*2} \approx 1$?
- Spearman Rank Correlation
- Variance-based, first-order effects is $\sum SI_{1st} \approx 1$?
- EASI, COSI
- Variance-based, higher-order effects is $\sum SI_{1st} + \sum SI_H \approx 1$?
- Variance-based on transformed data is $\sum SI^T_{1st} + \sum SI^T_H \approx 1$?
- Distribution/Density-based

No time left to address issues such as ...

- Time dependency
- Effects of Transformations on analysis results
- Input dependencies
- Thresholds / discontinuities

But bear in mind that there are also some conceptual issues ...

- What is the real purpose of SA?
 - ... to learn about the system?
 - ... to learn about the model?
 - ... to identify R&D needs?
 - ... to re-confirm / gain confidence?

- What if „conservative“ (rather than „best-estimate“) assumptions are used?
SA of any value in such cases?

- Why does almost no one study process / sub-system models?
Sensitivities with respect to function indicators?

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Thank you for your attention

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