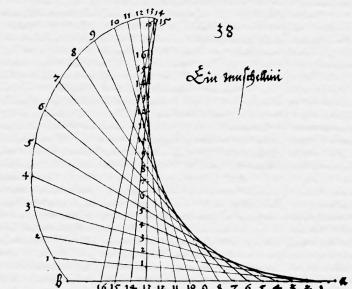


FAST COMPUTATIONAL METHODS FOR HANDLING LARGE SCALE DATA UNCERTAINTY PROBLEMS

S. Attinger, L. Schüler (UFZ Leipzig)
C. Reisinger, P. Schröder, G. Wittum



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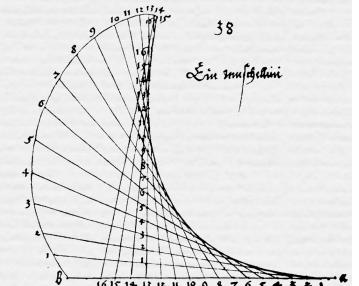


DATA UNCERTAINTY

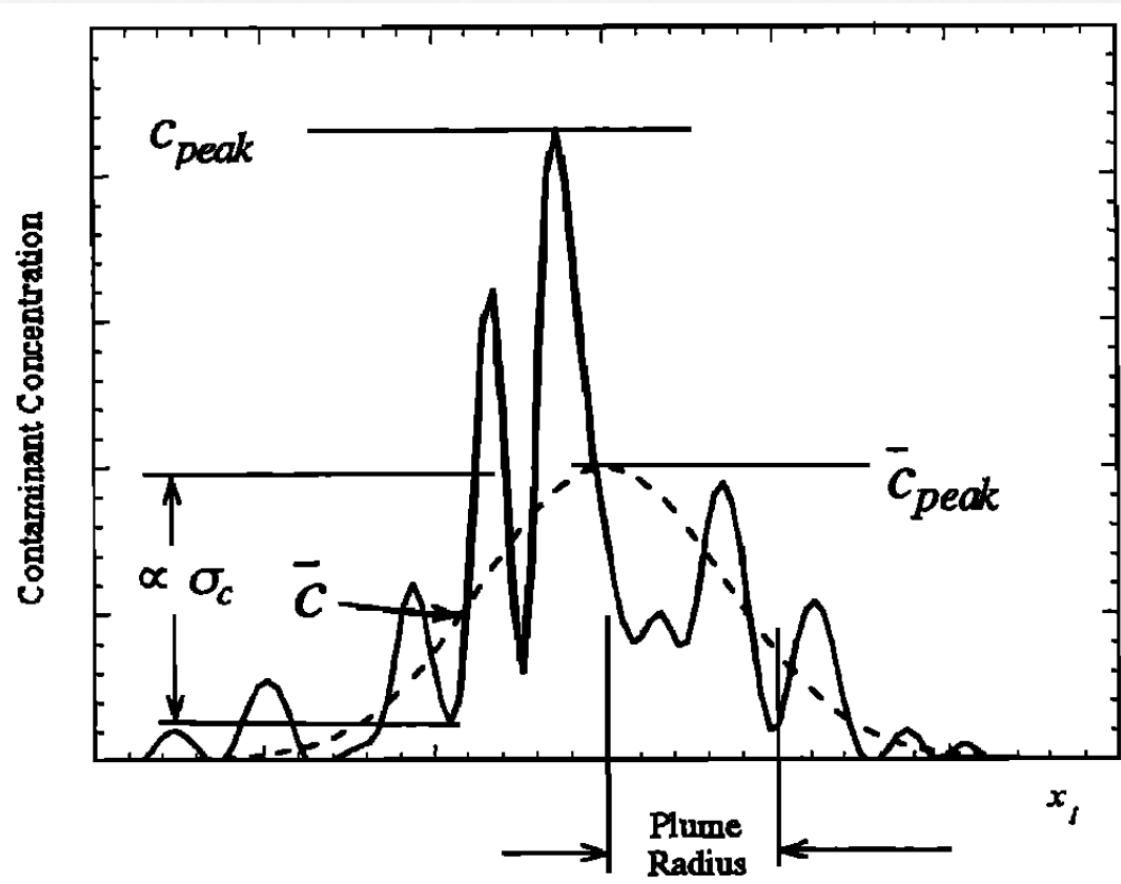
- In geological domains, only part of the structure is known, large parts remain unclear
- ...



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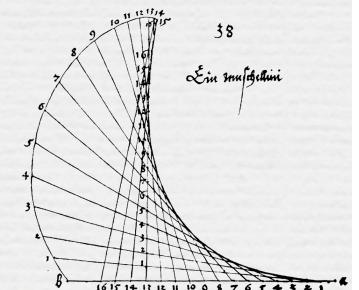
CONCENTRATION VARIATION



from Kapoor & Gelhaar, WRR 1994



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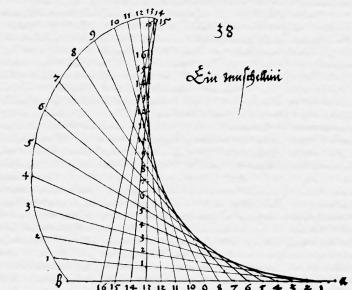


STANDARD MODELING APP.

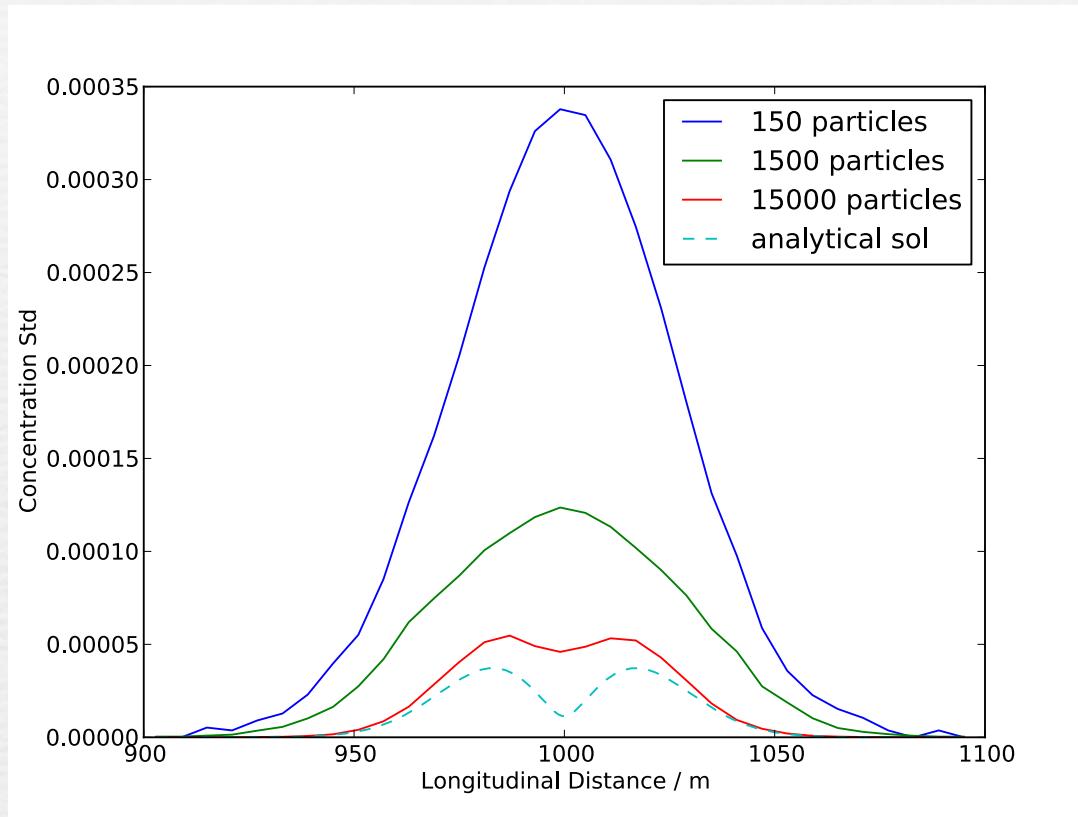
- Derive deterministic model
- Use Monte Carlo to account for stochastic variability
- Generate realizations satisfying the distribution assumption
- Run each realization and average them, record average and deviation



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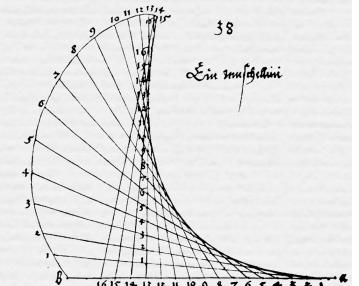
RANDOM WALK ALG.



- Computational costs forbidding



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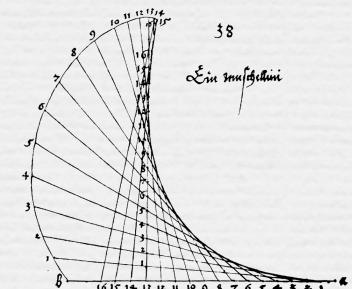


MATHEMATICAL VIEW

- Stochasticity means that we do not compute just one velocity, pressure and concentration. They vary among the realizations.
- Instead we need a probability density function (pdf) showing the distribution of the primary quantities.



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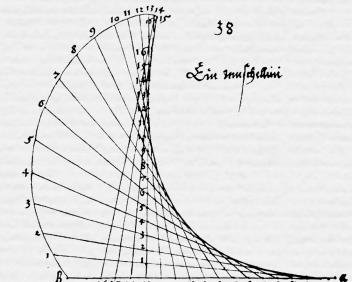
FOKKER PLANCK EQUATIONS

$$\begin{aligned}\frac{\partial}{\partial t} P(\vec{x}, t) = & - \sum_{i=1}^D \frac{\partial}{\partial x_i} (A_i(x_1, \dots, x_D) P(\vec{x}, t)) \\ & + \frac{1}{2} \sum_{i=1}^D \sum_{j=1}^D \frac{\partial^2}{\partial x_i \partial x_j} (B_{ij}(x_1, \dots, x_D) P(\vec{x}, t))\end{aligned}$$

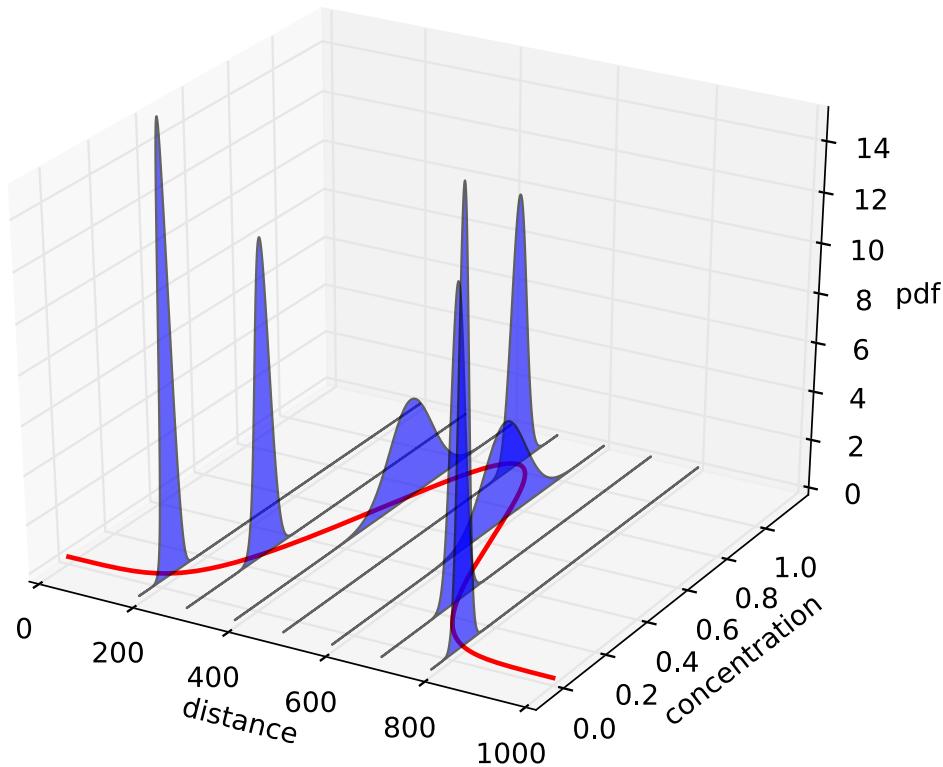
- 2nd order PDE, unknown PDF $P(x,t)$
- x comprises coordinates in space and primal variables
- D dimensions, $D > 3$



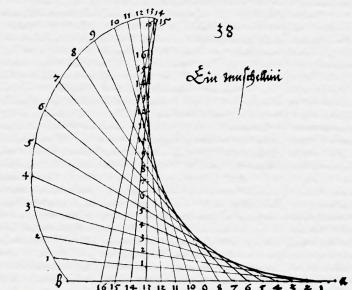
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SOLVING FP EQUATION

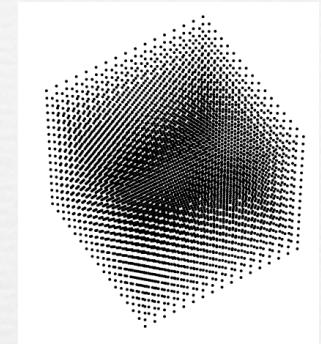


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HIGH DIMENSIONAL PDE

Work estimate



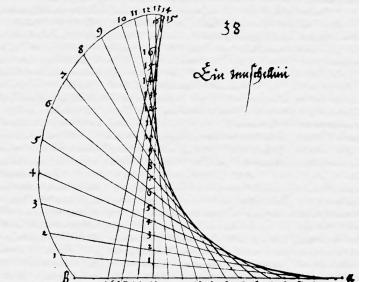
$$N(D, \epsilon) \sim \epsilon^{-D/p}$$

$$W(D, \varepsilon) \sim \varepsilon^{-D}$$

- **Curse of dimension!**



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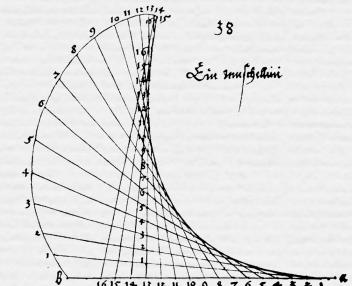
MONTE CARLO

$$W \sim \epsilon^{-2}$$

- Amount too big for a given problem due to the difficult choice of sampling points
- Fixed choice of sampling points (QMC) $\frac{(\log n)^d}{n}$,



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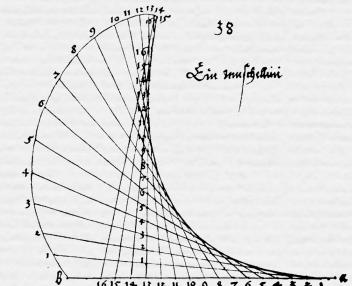


ALTERNATIVE

- Numerical simulation
- Make use of inherent structure of the problem



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DIMENSIONAL REDUCTION

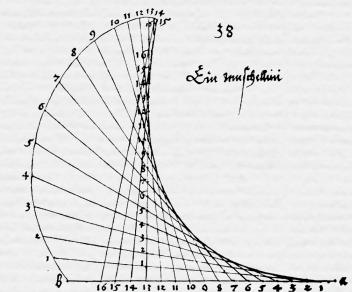
- Principal component analysis (PCA) of the covariance matrix $C = Q$

$$0 = \frac{\partial V}{\partial t} + \frac{1}{2} \sum_i \lambda_i \frac{\partial^2 V}{\partial z_i^2} - rV$$

with $Z = Q Y$



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ASYMPTOTIC EXPANSION

Taylor-like expansion of the solution in the eigenvalues of the covariance matrix (Reisinger, W., 2007)

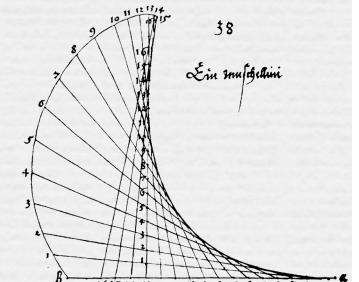
$$u(\mathbf{x}, t) = u^{(n)}(\mathbf{x}, t) + \sum_{j=n+1}^d \lambda_j \frac{\partial u}{\partial \lambda_j}(\mathbf{x}, t) \Big|_{\lambda^{(n)}} + \mathcal{O}(\lambda^{(n+1)})$$

where $u^{(n)}$

$$0 = \frac{\partial u}{\partial t} + \frac{1}{2} \sum_i \lambda_i^{(n)} \frac{\partial^2 u}{\partial x_i^2} - ru \quad \lambda_i^{(n)} := \begin{cases} \lambda_i & 1 \leq i \leq n, \\ 0 & \text{else} \end{cases}$$



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NUMERICAL RESULTS

Exact Solution

5D: 0.175864

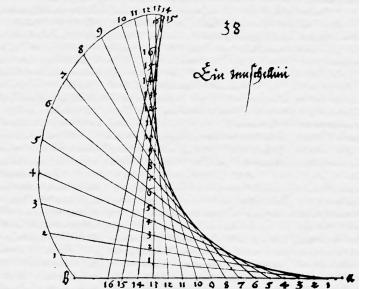
DAX: 0.135323

Taylor-like Expansion

| Order | Finance Basket | | DAX | |
|--------|----------------|-----------|----------|-----------|
| | result | rel error | result | rel error |
| 1st | 0.175803 | 0.043% | 0.134914 | 0.303% |
| 2nd | 0.175875 | 0.006% | 0.135306 | 0.013% |
| AE 2nd | 0.175813 | 0.03% | 0.13494 | 0.283% |



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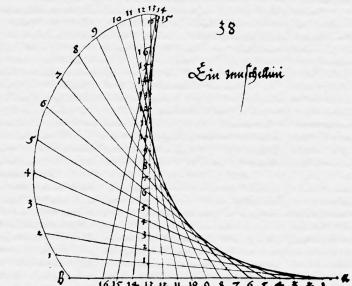


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- Schröder, P., Mlyzak, P., Wittum, G.: Dimension-wise Decompositions and their efficient Parallelization. In: Kloeden, P. (ed): Computational Finance, World Scientific, 2013.
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SUMMARY

- Fokker Planck eq. describes pdf directly
- 1 simulation instead of 1000 or more
- reduce high dimensional problems by asymptotic expansion (provided effective dimension $d \ll D$)
- apply fast algorithms in parallel
- Fokker Planck eq. can be solved in reasonable computing time



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