A Poroplastic Model of Bentonite Swelling

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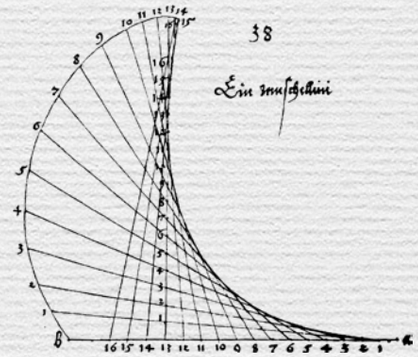

Available Models

- Elasto-plastic material laws
- standard algorithms can be used
- neglecting the fluid-structure interaction
- limited to small deformations
- no material reorganisation
- e.g.: Xie, Wang, de Jonge, Kolditz: Numerical Modelling of Swelling Pressure in Unsaturated Expansive Elasto-Plastic Porous Media. Transport in Porous Media (2007) 66:311–339

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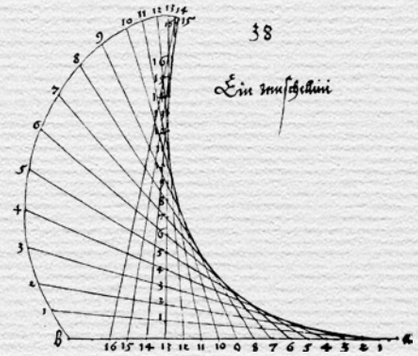

Swelling Model of Bentonite

New poroplastic model

Multiscale poroplasticity with material reorganisation

- Poroplastic material laws allowing
  - ✓ fluid-structure interaction
  - ✓ large deformations
  - ✓ inclusion of material reorganisation
- **multi-scale modeling approach allowing**
- **inclusion of design parameters (pellet size and form)**

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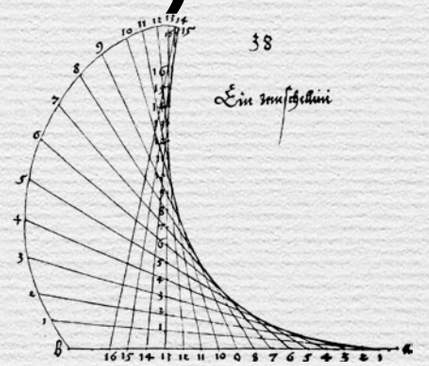

The Poroplastic Model

swelling-model of Bentonite

Mass-balance:

$$\dot{J} + \frac{1}{2} J_p \phi_{sn} \operatorname{tr}[\dot{\boldsymbol{B}}_\mathrm{p} \boldsymbol{B}_\mathrm{p}^{-1}] - \operatorname{Div}\left[\hat{\boldsymbol{K}}(\boldsymbol{F}, \boldsymbol{B}_\mathrm{p}) \operatorname{Grad} p\right] = 0\,,$$

Momentum-balance:

$$\operatorname{Div}\left(-Jp\boldsymbol{g}^{-1}\boldsymbol{F}^{-\mathrm{T}} + \hat{\boldsymbol{P}}_\mathrm{sc}(\boldsymbol{F}, \boldsymbol{B}_\mathrm{p})\right) = \boldsymbol{0}\,,$$

Evolution law:

$$\dot{\boldsymbol{B}}_\mathrm{p} + \hat{\boldsymbol{\mathcal{R}}}(\boldsymbol{F}, \boldsymbol{B}_\mathrm{p}) = \boldsymbol{0}$$

- with $\boldsymbol{F}$ the deformationgradient, $J = \det(\boldsymbol{F})$,
- with $\boldsymbol{B}_\mathrm{p}$ the inelastic deformation tensor, $J_p = \frac{1}{\sqrt{\det(\boldsymbol{B}_\mathrm{p})}}$,
- with p the pressure, $\hat{\boldsymbol{P}}_\mathrm{sc}$ the first Piola-Kirchhoff stress tensor,
- with $\hat{\boldsymbol{K}}$ the hydraulic conductivity tensor and ϕ_{sn} the solidity

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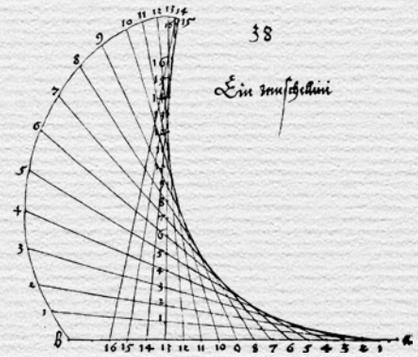

References

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B. Grillo, R. Prohl, G. Wittum: „***A poroplastic model of structural reorganisation in porous media of biomechanical interest***“, Continuum Mechanics and Thermodynamics, doi: 10.1007/s00161-015-0465-y (2015)

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A. Grillo, S. Federico, G. Wittum: **Growth, mass transfer, and remodeling in fiber-reinforced multi-constituent materials.** International Journal of Non-Linear Mechanics, Volume 47, Issue 2, Pages 388-401 (March 2012)

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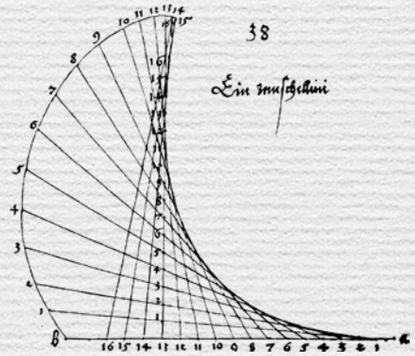
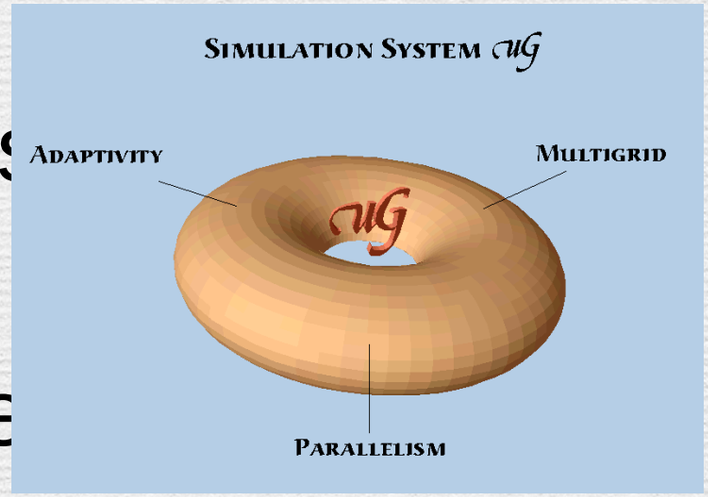


Simulation

- The Model
  - implemented in simulation system **≈4**
- **≈4** is a solver for general systems of pdes
  - unstructured grids,
  - multi-scale problems and method
  - parallel adaptive computing
  - excellent parallel scaling propertie
- software basis of **d³f** and **r³t**



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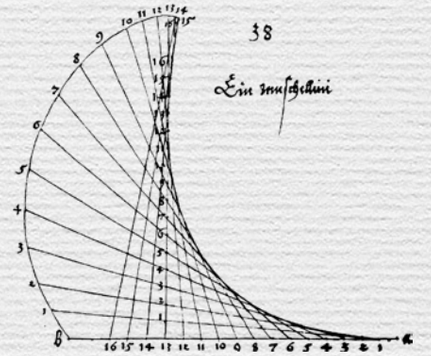
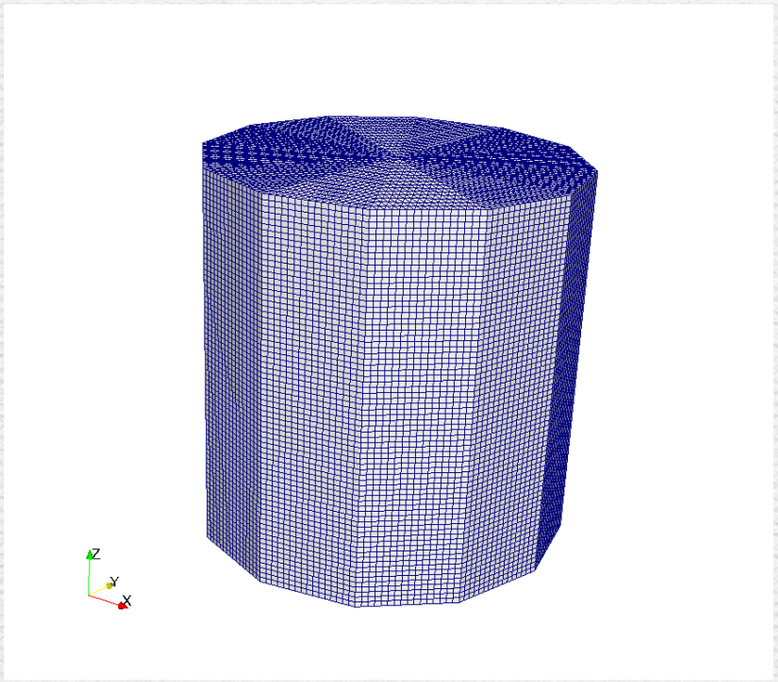


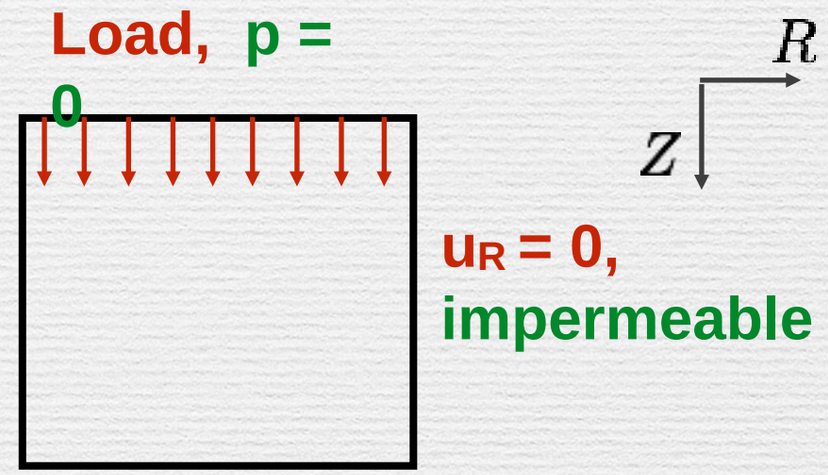
Swelling Test of Bentonite

Numerical Test

Oedometer test of a cylinder-specimen (mentioned e.g. in Nguyen et al, 2004):

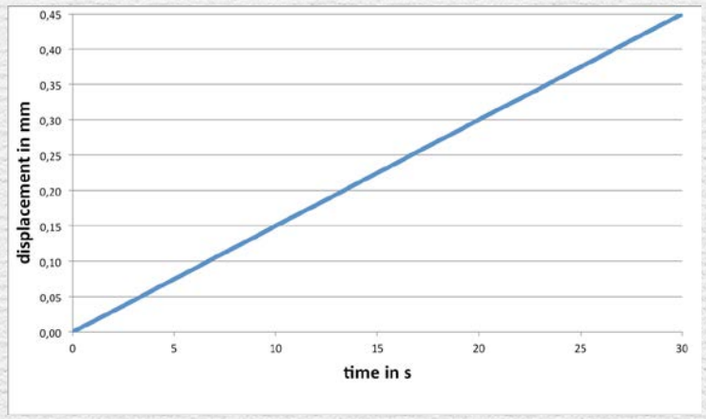


Boundary conditions (solid, fluid):



height (H_0): 3 mm, diameter: 3 mm

Load (Compression):



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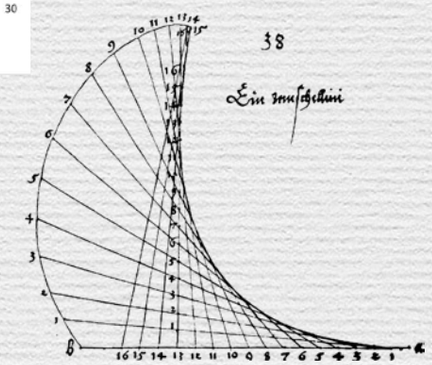
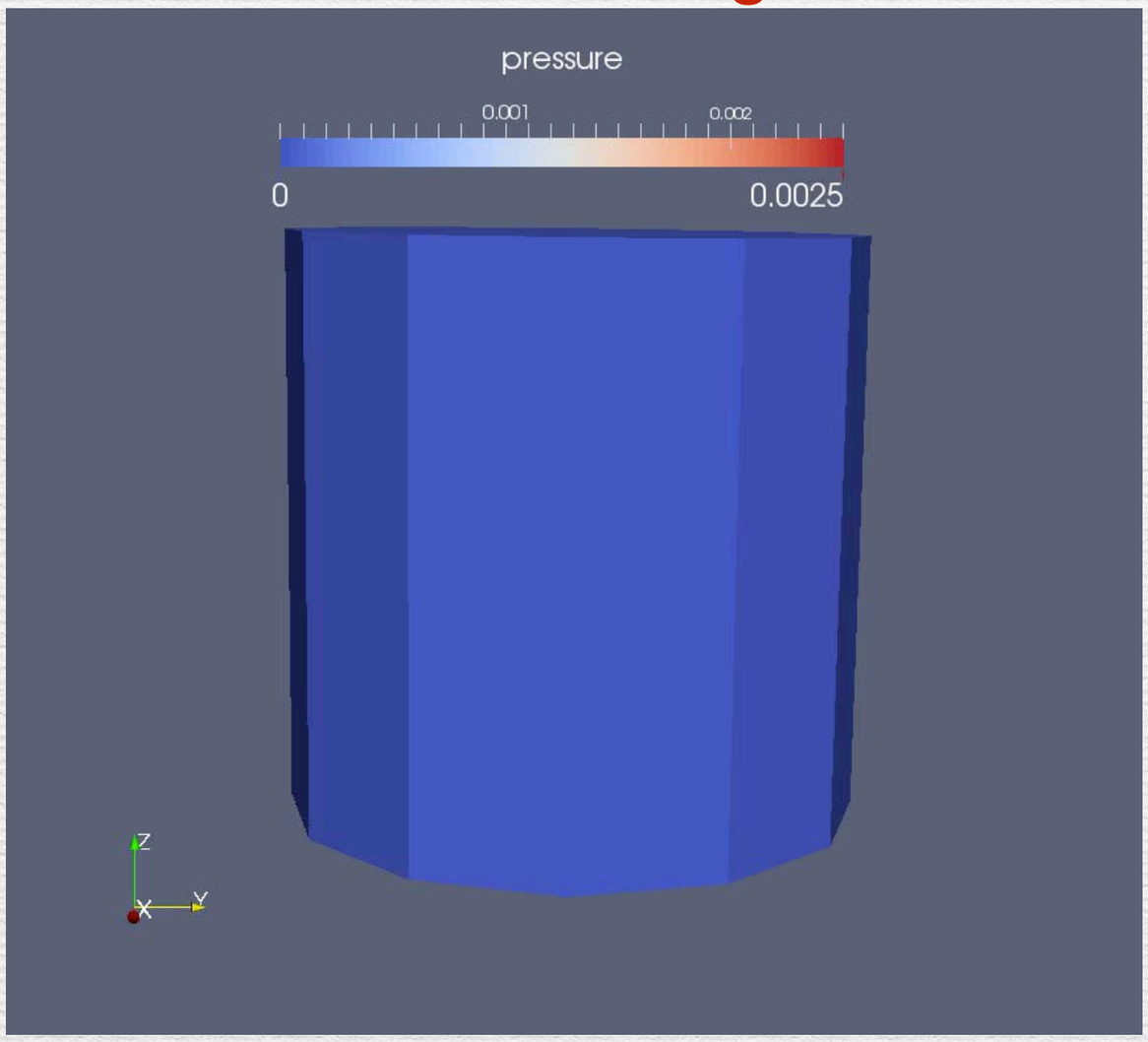


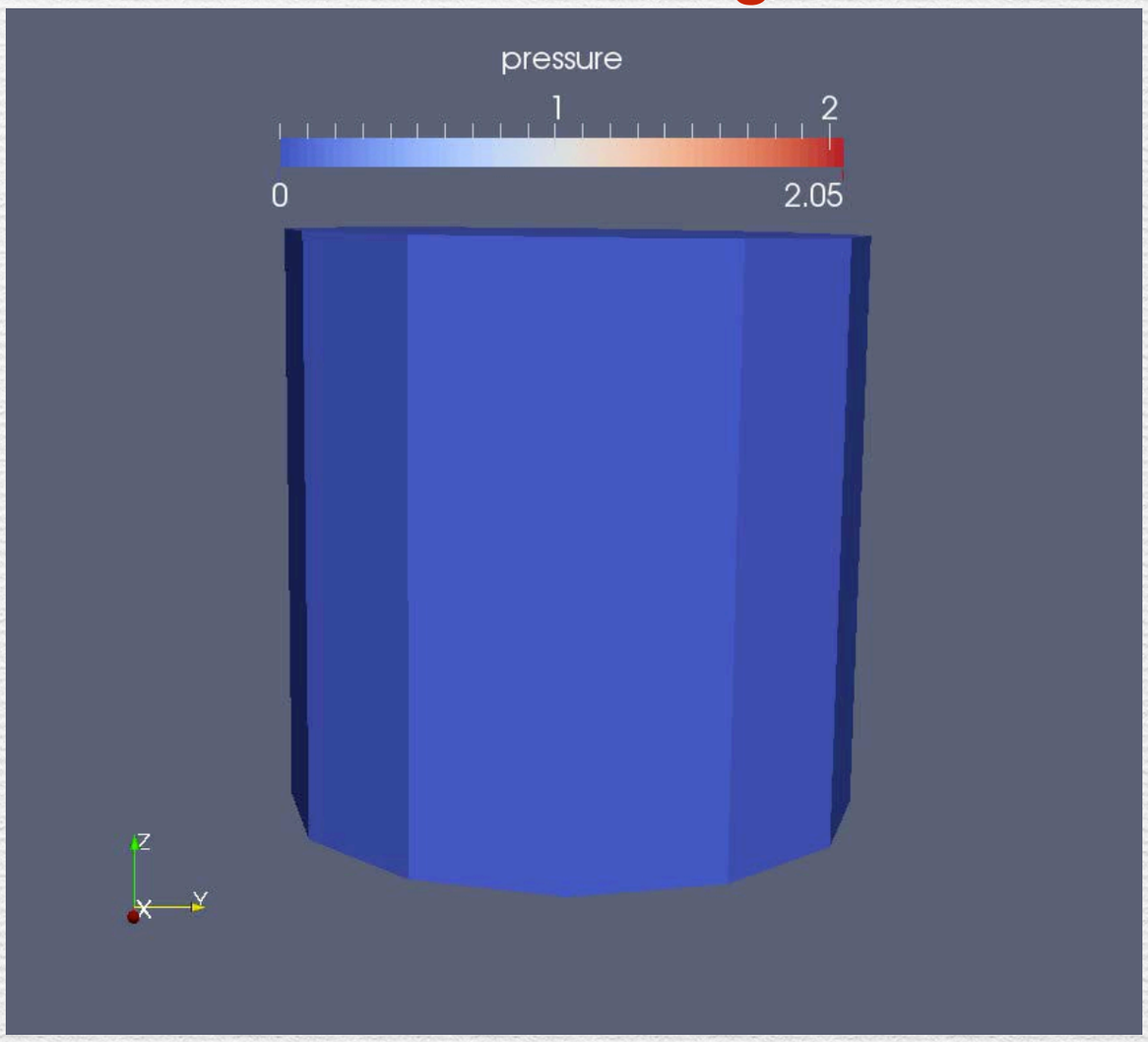
Comparison of the Pressure

Swelling Test of bentonite

without swelling



with swelling



→ **swelling leads to larger pressure in the cylinder**

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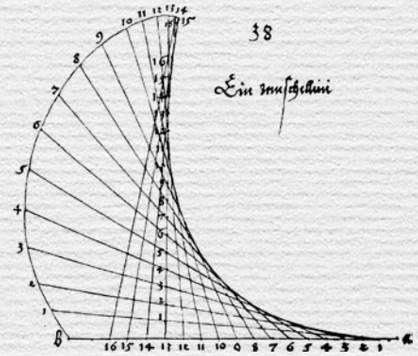

Proposal

New poroplastic model

Include poroplastic multiscale model into the new project

- **This will allow**
- **description of the bentonite reorganisation process**
- **inclusion of design parameters (pellet size and form)**
- **inclusion of anisotropies e.g. due to construction layers**

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